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Neutrino masses and mixings from string theory instantons

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Abstract: We study possible patterns of neutrino masses and mixings in string models in which Majorana neutrino masses are generated by a certain class of string theory instantons recently considered in the literature. These instantons may generate either directly the dim=5 Weinberg operator or right-handed neutrino Majorana masses, both with a certain flavour-factorised form. A hierarchy of neutrino masses naturally appears from the exponentially suppressed contributions of different instantons. The flavour structure is controlled by string amplitudes involving neutrino fields and charged instanton zero modes. For some simple choices for these amplitudes one finds neutrino mixing patterns consistent with experimental results. In particular, we find that a tri-bimaximal mixing pattern is obtained for simple symmetric values of the string correlators.

Keywords: Intersecting branes models, Neutrino Physics, Superstring Vacua, Nonperturbative Effects.

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1. Introduction

Recently a new mechanism for the generation of neutrino Majorana masses in the context of string theory has been pointed out [1 − 4]. Certain string instanton effects can generate right-handed neutrino masses from operators of the form

$$e^{-U_M}\,\nu_R\nu_R M_{\rm string}\ . \tag{1.1}$$

Here M_s is the string scale and U_M is a complex scalar modulus field whose axion-like imaginary part $Im\,U_M$ gets shifted under a gauged $\mathrm{U}(1)_{B-L}$ symmetry in such a way that the operator (1.1) is $\mathrm{U}(1)_{B-L}$ gauge invariant. The size of these masses is of order $\exp(-Re\,U_M)M_s$. Unlike ordinary, e.g., electroweak instanton effects which are of order $\exp(-1/\alpha_2)$, these instantons need not be very much suppressed, $Re\,U_M$ is not the inverse of any SM gauge coupling and may be relatively small. Thus right-handed neutrino masses may be large, as required phenomenologically. Furthermore it was noted [4] that analogous instantons can also generate a dimension 5 Weinberg operator of the form

$$e^{-U_W}\,\frac{1}{M_s}\overline{H}L\overline{H}L\ . \tag{1.2}$$

This term gives rise directly to left-handed neutrino masses once the Higgs scalars get a vev. Both these instanton effects only appear in a restricted class of string compactifications in which the SM gauge group is extended by a $\mathrm{U}(1)_{B-L}$ gauge boson which is massive through a Stuckelberg mass term. String compactifications in which such instanton mechanism is operative have been recently discussed in [1–4].

In the present paper we make a first phenomenological exploration of the structure of neutrino masses and mixings obtained from this string instanton mechanism. In this analysis we will concentrate on a particular class of instantons, those with internal Sp(2) Chan-Paton (CP) symmetry which leads to the simplest structure and appear most often in available instanton searches [4]. For such instantons the flavour dependence of both ν_R-masses and the Weinberg operator factorises as product of flavour vectors (called d_a and c_a (a=1,2,3) in the main text for the ν_R-masses and Weinberg operator respectively). These flavour vectors d_a, c_a may be in principle computable in terms of the specific underlying string compactification. This simple flavour structure and the fact that one expects several different instantons contributing to the amplitude make it quite natural to obtain a hierarchy of neutrino masses [4].

The structure of this paper is as follows. In the next section, section 2, we present a brief overview of the string instanton mechanism which is relevant for the generation of neutrino Majorana masses. We discuss how the operators in eqs. (1.1) and (1.2) may be generated as well as their flavour structure and the expected size of the neutrino masses. Turning to the phenomenological analysis in section 3, in section 3.1 we consider the case in which the Weinberg operator is dominant compared to the see-saw contribution. We also assume in a first approximation that the large mixing in the leptonic sector originates in the neutrino mass matrix (and not in the charged leptons). In this case the physical neutrino mass matrix is directly obtained from the discussed instanton effects and the analysis is much easier. We show that, if there is a hierarchy of neutrino masses (naturally induced by the above-named instanton effects), then one can obtain a neutrino mixing matrix consistent with experimental results for certain (not very stringent) constraints on the values of the flavour vectors c_a. We also show that for flavour vectors c_a along particular directions one can reproduce, e.g., tri-bimaximal mixing both for the normal and inverse hierarchy cases.

We then consider the case in which the see-saw contribution to neutrino masses is dominant in section 3.2. In this case the final result for the physical neutrino masses depends on the structure of the Dirac mass for the neutrinos. This makes the analysis more model-dependent. We consider a simple case in which the Dirac mass matrix is diagonal. In this case one can obtain e.g. tri-bimaximal mixing if the flavour vector coefficients d_a of different contributing instantons align along certain directions in flavour space. The case in which both the Weinberg operator and see-saw mechanism are relevant is briefly discussed in section 3.3. Some final conclusions and some comments are left to section 4.

2. Neutrino masses and string instanton effects

In large classes of string compactifications the gauge group of the SM fields includes an extra $\mathrm{U}(1)_{B-L}$ gauge interaction. This is to be expected since $\mathrm{U}(1)_{B-L}$ is the unique flavour-independent U(1) symmetry which is anomaly free (in the presence of three right-handed neutrinos ν_R^a, needed also for the cancellation of mixed $\mathrm{U}(1)_{B-L}$-gravitational anomalies).

In fact practically all semi-realistic MSSM-like string compactifications constructed up to now have such an extra $U(1)_{B-L}$ interaction and three right-handed neutrinos.

We will focus on this class of string compactifications with an extra $U(1)_{B-L}$. Of course, such a gauge interaction forbids the presence of Majorana masses for neutrinos, since they would violate $U(1)_{B-L}$ gauge invariance. However, as pointed out in [1] (see also [2]), string instanton effects may give rise to right-handed neutrino Majorana masses under certain conditions. In particular a crucial point is that the $U(1)_{B-L}$ gauge boson should get a Stuckelberg mass from a $B \wedge F$ type of coupling. Here B is a 2-index antisymmetric field[1] and F is the $U(1)_{B-L}$ field strength. This mechanism is ubiquitous in string theory and it plays an important role in $U(1)$ anomaly cancellation by the Green-Schwarz mechanism (for a simple discussion see e.g. [5]). Due to the presence of the $B \wedge F$ coupling, the pseudo-scalar η (dual to the B field) transforms under a $U(1)_{B-L}$ gauge transformation of parameter $\Lambda(x)$ as:

$$\eta(x) \ \longrightarrow \ \eta(x) \ + \ q \ \Lambda(x) \ , \tag{2.1}$$

with q being some integer. The η scalar has then a Higgs-like behavior and gives a mass of order the string scale M_s to the $U(1)_{B-L}$ gauge boson. Thus, from the low-energy point of view the gauge symmetry is just that of the SM (or possibly e.g. a SU(5) extension).

As pointed out in [1, 2] in this class of models string instantons can give rise to terms of the form

$$W_2 \simeq e^{-U_{\rm ins}} \, \nu_R \nu_R \ , \tag{2.2}$$

which give rise the right-handed neutrino Majorana masses. Here $U_{\rm ins}$ is a complex modulus scalar field characteristic of the instanton and the particular compactification. The point is that $Im\, U_{\rm ins}$ is a linear combination of axion-like fields including η in such a way that under a $U(1)_{B-L}$ gauge transformation transforms like

$$Im\, U_{\rm ins}(x) \ \longrightarrow \ Im\, U_{\rm ins}(x) \ - \ 2\Lambda(x) \ . \tag{2.3}$$

Then the operator $\exp(-U_{\rm ins})$ has effective B-L charge=2 and the operator (2.2) is gauge invariant, the gauge transformation of the neutrino bilinear is canceled by the exponential.

This type of instanton contributions may appear in all 4-dimensional string constructions but it is particularly intuitive in the case of Type IIA orientifolds with intersecting D6-branes [6 − 8] (see e.g. [9] for reviews and references). These D6-branes have a 7-dimensional worldvolume including Minkowski space. The remaining 3-dimensions wrap a 3-cycle Π of volume V_Π in the 6 compact dimensions. In these models quarks and leptons appear as string excitations localised at D6-brane intersections. In the simplest configurations there are 4 different stacks of such D6-branes a,b,c,d associated to gauge groups $U(3)_a \times SU(2)_b \times U(1)_c \times U(1)_d$. The $U(1)_{a,d}$ gauge symmetries correspond to baryon and lepton number respectively and $U(1)_c$ may be identified with the diagonal generator of right-handed weak isospin. Out of these 3 $U(1)$'s only the linear combination

[1] These tensors come e.g. from the RR-sector of Type II string theory. In $D = 4$ they are dual to pseudo-scalar fields η_r which are the imaginary part of complex scalar moduli fields, either complex structure moduli U_r or Kahler moduli T_r, depending on the specific compactification.

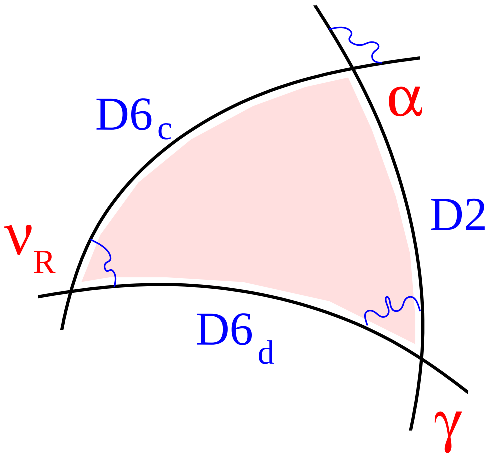

Figure 1: World-sheet disk amplitude inducing a cubic coupling on the D2-brane instanton action. The cubic coupling involves the right-handed neutrinos lying at the intersection of the c and d D6-branes, and the instanton fermion zero modes α and γ from the D2-D6 intersections.

$Y = Q_a/6 - Q_c/2 - Q_d/2$ corresponding to hypercharge remains light. The linear combination $3Q_a + Q_d$ has triangle anomalies and gets a Stuckelberg mass as usual. The remaining orthogonal linear combination $Y' = Q_a/6 + Q_c/2 - Q_d/2$ is anomaly-free and again gets a Stuckelberg mass. The $\mathrm{U}(1)_{B-L}$ generator is given by $Y + Y'$.

In these intersecting D6-brane models string instantons [10, 11] are D2-branes with their 3-dimensional volume wrapping a 3-cycle Π_M on the 6 extra dimensions. This is just like D6-branes, the main difference being that these D2-branes are localised in $D = 4$ space and time (that is why they are identified with instantons). The action of these instantons is just the D2-brane classical action, which is given by the Born-Infeld action which yields[2]

$$S_{D2} = \frac{V_\Pi}{\lambda} + i \sum_r q_{M,r} \eta_r \ , \tag{2.4}$$

where V_Π is the 3-volume wrapped by the D2-brane, λ is the string dilaton and the imaginary piece is a linear combination with integer coefficients of axion-like RR-fields characteristic of the particular instanton M. For any given compactification and instantons S_{D2} may be written as a particular linear combination of moduli fields $S_{D2} = U_{\rm ins}$. In the particular case of Type IIA orientifolds with intersecting D6-branes, they are complex-structure moduli of the compact manifold.

As described in [1, 2, 4] there is in fact an extra contribution to the instanton action which comes from possible intersections of the D2-instanton and the relevant c and d D6-branes (see figure 1 for a pictorial view). Right-handed neutrinos come from string excitations around the intersections of d and c branes. On the other hand the D2-instanton may intersect with the d and c branes and at their intersections string excitations give rise to fermionic zero modes α_i and γ_i. We will soon see that for a ν_R bilinear to be generated the

[2]In the present discussion we will always work in string mass units with $M_s^2 = (\alpha')^{-1} = 1$, recovering the string mass dimensions for the final neutrino formulae.

multiplicity of these modes must be two, i.e., $i, j = 1, 2$. Note that, since D2-branes do not include Minkowski space inside their volume, α_i, γ_i are not $3+1$ dimensional particles, like the ν_R's but rather 0+0 dimensional zero modes. They will behave like Grassman variables over which one has to integrate. Indeed, in computing the contribution of instantons to a given amplitude (both with standard YM instantons and with string instantons of the type here considered) one has to integrate over the moduli of the instanton and these α_i, γ_i zero modes will be part of it. Now, there are non-vanishing amplitudes among the right-handed neutrinos ν_R^a and the zero modes which contribute to the instanton action

$$S_{\rm ins}(\alpha,\gamma) \;=\; d_a^{ij}\;(\alpha_i\;\nu^a\gamma_j)\;, \quad a=1,2,3\;. \tag{2.5}$$

Here d_a^{ij} are coefficients which depend on the Kahler moduli of the compactification. In order to obtain the induced superpotential one has to integrate over the fermionic zero modes α_i, γ_i and one obtains a superpotential coupling[3] proportional to [1, 2, 4]

$$\int d^2\theta \int d^2\alpha\, d^2\gamma\; e^{-d_a^{ij}\;(\alpha_i\nu^a\gamma_j)} \;=\; \int d^2\theta\, \nu_a\nu_b\,(\,\epsilon_{ij}\epsilon_{kl}d_a^{ik}d_b^{jl}\,)\;, \tag{2.6}$$

where we have made use of the Grassman integration rules $\int d\alpha = 0$, $\int d\alpha\alpha = 1$ etc. Note that the fact that we have two α and γ zero modes is crucial in order to obtain a bilinear. This expression is multiplied by the exponential of the classical action (2.4) so that the final expression for the right-handed neutrino Majorana mass has the form

$$M_{ab}^R \;=\; M_s(\,\epsilon_{ij}\epsilon_{kl}d_a^{ik}d_b^{jl}\,)\exp(-U_{\rm ins})\;, \quad a,b=1,2,3\;, \tag{2.7}$$

where M_s is the string scale and ϵ_{ij} is the 2-index antisymmetric unit tensor. Note that the flavour information is encoded in the couplings d_a^{ij}. As discussed in detail in [4], the relevant D2-instantons have a gauge symmetry which is realised only as a global symmetry in the effective $D = 4$ spectrum. The simplest and most frequent situation found up to now is that the global symmetry is Sp(2) = SU(2), so that the α and γ zero modes are doublets of SU(2). In that situation one can write $d_a^{ij} = d_a\epsilon^{ij}$ and the Majorana mass matrix takes a factorised form

$$M_{ab}^R \;=\; 2M_s\sum_r d_a^{(r)}d_b^{(r)}\; e^{-U_r}\;, \tag{2.8}$$

where the sum goes over the different instantons which may contribute to this Majorana mass term (in general there are several different instantons contributing). As noted in [4], this expression has an interesting flavour structure. Indeed one can write

$$M^R \;\sim\; \sum_r\; e^{-U_r}{\rm diag}\,(d_1^{(r)},d_2^{(r)},d_3^{(r)})\cdot\begin{pmatrix}1\;1\;1\\1\;1\;1\\1\;1\;1\end{pmatrix}\cdot{\rm diag}\,(d_1^{(r)},d_2^{(r)},d_3^{(r)})\;. \tag{2.9}$$

[3] There are additional factors of order one coming from the quantum fluctuations of massive modes (see e.g. [2, 12]) . We set those terms to one in the present analysis.

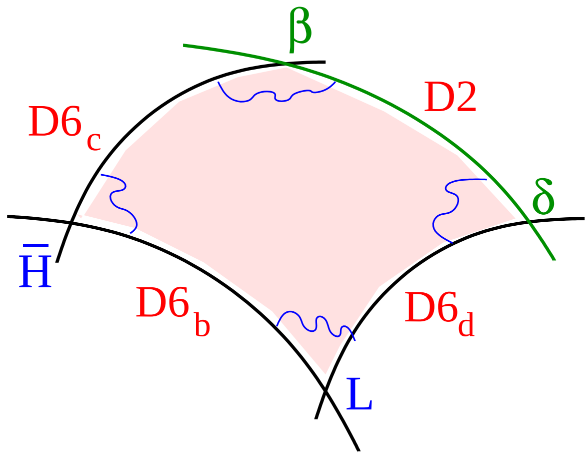

Figure 2: World-sheet disk amplitude inducing a quartic coupling on the D2-brane instanton action. The coupling involves the Higgs $\overline{H}$ and left-handed leptons L^a lying at the intersection of the b, c and d D6-branes, and the instanton fermion zero modes β and δ from the D2-D6 intersections.

With this structure each instanton makes one particular linear combination of ν_R' s massive, leaving two linear combinations massless. In particular one(two) instanton(s) contribution(s) would leave two(one) neutrinos massless. Thus with three or more contributing instantons generically all three get a mass. Furthermore a hierarchy among the three eigenvalues may naturally appear taking into account that each instanton will have in general a different suppression factor $\exp(-Re\, U_r)$. This will be one of the crucial ingredients of our phenomenological analysis in the next sections.

Once (large) right-handed neutrino masses are generated the standard see-saw mechanism [13] is expected to induce Majorana masses for the lightest eigenvalues in the usual way, i.e. neutrino masses of the form

$$M^L_{ab}(\text{see-saw}) \;=\; \frac{\langle \overline{H} \rangle^2}{2M_s} h_D^T (\sum_r \; d_a^{(r)} d_b^{(r)} \; e^{-S_r})^{-1} \; h_D \;, \tag{2.10}$$

where h_D is the ordinary Yukawa coupling constant in $h_D^{ab}(\nu_R^a \bar{H} L^b)$. The eigenvalues of these matrices are the ones which should be compared with experiment.

As we mentioned, there is a second lepton number violating operator which can be relevant for the structure of neutrino masses. This is the dim=5 Weinberg operator (in superpotential form)

$$W_W \;=\; \frac{\lambda_{ab}}{M} (L^a \overline{H} L^b \overline{H}) \;. \tag{2.11}$$

Once the Higgs fields get a vev, left-handed neutrino masses of order $\langle \bar{H} \rangle^2 \lambda_{ab}/M$ are generated. One important aspect of this operator is that it does not involve any field beyond those of the SM (not even ν_R' s) and does not directly involve the see-saw mechanism. String instantons can again give rise to such a superpotential in the class of string models under consideration. A superpotential of the form

$$W_W \;=\; e^{-U_{\rm ins}}\frac{\lambda_{ab}}{M_s}(L^a\overline{H}L^b\overline{H})\ . \tag{2.12}$$

may be generated in a way totally analogous to the one discussed above for the ν_R Majorana masses. The only main difference is that this time the corresponding D2-instantons (which are different from the ones giving rise to ν_R masses) have zero modes β_i, δ_i with quartic couplings

$$S_{\rm ins}(\beta,\delta) \;=\; c_a^{ij}\ (\beta_i(L^a\overline{H})\delta_j)\ . \tag{2.13}$$

Again, in the simplest case with a SU(2) symmetry operating in the i, j indices one has $c_a^{ij} = c_a\epsilon^{ij}$ and a flavour factorised expression is obtained for the left-handed neutrino Majorana masses

$$M^L_{ab} \;=\; \frac{2\left\langle\overline{H}\right\rangle^2}{M_s}\sum_r c_a^{(r)}c_b^{(r)}\ e^{-U_r}\ , \tag{2.14}$$

where again the sum runs over possible different instantons contributing. Note that the flavour structure of this mass matrix is the same as that of eq. (2.9) so that a hierarchy of neutrino masses naturally appears.

A comment is in order concerning the relationship between the see-saw mechanism and the dim=5 Weinberg operator. For constant field-independent Majorana ν_R masses, the exchange of the ν_R fields gives rise to a see-saw superpotential contribution to the Weinberg dim=5 term. On the other hand for field dependent masses like those generated from instantons, one cannot write down the see-saw contribution in the form of a Weinberg holomorphic superpotential. So both contributions should be considered separately and in fact in the string models different instantons contribute to both effects [4].

In a given string compactification both mechanisms (see-saw and direct dim=5 operator) may be present. Which is the dominant effect concerning the determination of the masses and mixings of the observed neutrinos will be model dependent. In particular it will depend on the particular values of the instanton actions $Re\,U_{\rm ins}$, the value of the string scale M_s, the values of the coefficients c_a, d_a and of the neutrino Yukawa couplings h_D. In particular, if the h_D couplings are small and there is little suppression from the exponential factors $\exp(-U_r)$, the Weinberg operator might be dominant. As we will see this case is particularly simple because then one can directly correlate the neutrino flavour structure to the string instanton mass generation formulae. In the see-saw case the dependence on the string instanton effects is partially masked by the dependence on the flavour dependent h_D Yukawa couplings.

Looking at formulae (2.8) and (2.14) we see that the obtained neutrino masses depend on three quantities, the string scale M_s, the instanton actions $Re\,U_r$ and the instanton couplings c_a or d_a. Before entering into the phenomenological analysis of the following sections let us review what can be said about these quantities. Concerning the string scale M_s, it is in principle undetermined by present data and may be as small as the TeV scale. On the other hand if we want to keep gauge coupling unification and other simple features of MSSM-like scenarios, identifying M_s with the GUT-scale (i.e. of order

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10^{16} GeV) is an attractive option. The Weinberg operator may induce neutrino masses of order $10^{-1} - 10^{-2}$ eV as long as

$$M_s \ < \ 10^{15} (c_a)^2 e^{-Re\, U_W} \text{ GeV} \ . \tag{2.15}$$

If this was the dominant source of neutrino masses, this would seem to favor values of the string scale below the unification scale 10^{16} GeV. However, if there are a number of different instantons contributing and $Re\, U_W$ is small, it could still be computable with M_s of order the unification scale.

If the dominant source of observed neutrino masses were the see-saw mechanism, one can obtain neutrino masses of order $10^{-1} - 10^{-2}$ eV as long as

$$M_s \ < \ 10^{15} \frac{h^2}{d_a^2} e^{Re\, U_M} \text{ GeV} \ , \tag{2.16}$$

where h is the size of the largest neutrino Yukawa coupling. In this case the size of the string scale is essentially unconstrained.

Concerning the values of the string instanton actions $Re\, U_r$, it is important to remark that, unlike the standard YM instantons of electroweak interactions, string instanton effects are not particularly suppressed, since $Re\, U_r$ are unrelated to the (inverse) gauge couplings of any SM interactions. This means that the exponential factors $\exp(-Re\, U_r)$ appearing in the amplitudes may be in fact of order one or, say, $O(1/10)$ and hence the neutrino operators we are talking about need not be particularly suppressed. The actions of each individual instanton, proportional to $Re\, U_r$, are generically different. For example, in section 6.4 of [4] an example of compactification is shown in which there are three instantons contributing to right-handed neutrino masses whose actions are on the ratios $1 : 3.8 : 16.2$. The overall normalization depends on the value of the Type II string dilaton, which is a free parameter in perturbative compactifications. This example illustrates how indeed a hierarchy of neutrino mass eigenvalues is possible in the present context.

Concerning the amplitudes c_a and d_a, $a = 1, 2, 3$, they are obtained from string correlators involving the chiral field operators $\bar{H}L^a$ and ν_R^a respectively and the fermionic instanton zero modes β_i, δ_i and α_i, γ_i, $i = 1, 2$. In the case of intersecting D6-brane models they are in general functions of the Kahler moduli T_k of the compactification. If the string compactification involves a known conformal field theory (CFT) like in toroidal (or orbifold) models or models obtained from Gepner orientifolds [14] (see also [4] and references therein), c_a,d_a are computable in principle. In practice only for toroidal models such computations are available at the moment (very much like it happens with ordinary Yukawa couplings). However, although there have been constructed Type IIA orbifold orientifolds with intersecting D6-branes and a MSSM-like spectrum (see [9] for reviews and references) none of them have a massive $\mathrm{U}(1)_{B-L}$ as required for the present instanton mechanism to work. On the other hand there are non supersymmetric intersecting D6-brane models in which such massive $\mathrm{U}(1)_{B-L}$ gauge bosons occur. As pointed out in [1], for such models the $d_a(T_k)$ amplitudes are analogous to those of ordinary Yukawa couplings [15] and they are typically proportional to (products of) Jacobi theta functions $\theta[\delta^a, 0](\phi_i, T_k)$. Here δ^a are some fractional numbers which depend on the generation number $a = 1, 2, 3$,

T_k are Kahler moduli and ϕ_I are scalar moduli fields which parameterise the location of the D-branes in extra dimensions. However, the non-SUSY toroidal examples discussed in [1] need to be completed since they require the presence of further backgrounds in order to get the adequate number of instanton zero modes for the neutrino mass operators to be generated. Still this at least illustrates what type of functions could appear in more realistic computations. For example, we will see in the phenomenological applications in the next section that, e.g., in order to have a small θ_{13} in the neutrino mixing matrix, a suppressed c_1 amplitude for the leading instanton would be required. So we might want to impose such a condition on candidate string compactifications. In this connection it is perhaps worth noticing that Jacobi theta functions do vanish in particular symmetric points.

Given our discussion above, our approach in the present paper concerning the amplitudes c_a, d_a will be mostly phenomenological. We will address ourselves the question: under what circumstances are the present class of instanton induced neutrino masses consistent with present experimental constraints? In the next section we will see that under very mild constraints on the c_a coefficients the instanton generated Weinberg operator will be consistent with present experimental data on neutrino masses and mixings. Furthermore we will see that if the c_a amplitudes go along certain directions in flavour space, e.g., tri-bimaximal neutrino mixing may be obtained.

3. Neutrino masses and mixing angles

In general, the leptonic mixing matrix (PMNS matrix), is given in the standard PDG parameterisation as

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P_{\text{Maj}} \,, \tag{3.1}$$

where we have used the abbreviations $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$. Here δ is the so-called Dirac CP violating phase which is in principle measurable in neutrino oscillation experiments, and $P_{\text{Maj}} = \text{diag}(e^{i\frac{\alpha_1}{2}}, e^{i\frac{\alpha_2}{2}}, 0)$ contains the Majorana phases α_1, α_2. The PMNS mixing receives contributions from the matrix V_{e_L} diagonalizing the mass matrix of the charged leptons and from V_{ν_L} diagonalizing the neutrino mass matrix,

$$U_{\text{PMNS}} = V_{e_\text{L}} V^\dagger_{\nu_\text{L}} \,. \tag{3.2}$$

In the following, we assume that the large mixing in the lepton sector originates from the neutrino mass matrix. In fact such large mixings are generically expected in the present context, given the very different origins of the neutrino masses from string instantons, compared to the masses for charged leptons and quarks. Under this assumption, we may treat the small mixings of the charged lepton mass matrix as a perturbation.

3.1 Masses and mixings from the Weinberg operator

We first discuss the case that the contribution from the dimension 5 Weinberg operator dominates the neutrino mass matrix. As we said, in this case the left-handed neutrino mass matrix is directly related to the instanton contribution discussed in the previous section.

3.1.1 The general normal hierarchy case

When the Weinberg operator dominates, the instanton-induced neutrino mass matrix can be written in the form

$$M^L_{ij} = \sum_r I_r c_i^{(r)} c_j^{(r)}, \tag{3.3}$$

where we have defined

$$I_r = \frac{\langle H \rangle^2}{M_s} 2 e^{-S_r} \, . \tag{3.4}$$

Let us consider first the scenario in which we have three instanton contributions to the neutrino mass matrix ($r = 1, 2, 3$). Let us assume there is some (eventually mild) hierarchy, in particular

$$|I_3 c_i^{(3)} c_j^{(3)}| \gg |I_2 c_i^{(2)} c_j^{(2)}| \gg |I_1 c_i^{(1)} c_j^{(1)}| \, . \tag{3.5}$$

This may be motivated by a hierarchy of the instanton factors,

$$e^{-Re(S_3)} \gg e^{-Re(S_2)} \gg e^{-Re(S_1)} \, , \tag{3.6}$$

As we mentioned in the previous section, such modest hierarchies are likely in orientifold compactifications (see e.g. [4]). In this situation we can extract analytically the conditions under which the generated neutrino masses and mixing angles are compatible with the experimental results.

In order to simplify the following discussion of the leptonic mixing angles and CP phases resulting from neutrino masses of the form in eq. (3.3), we define

$$\bar{c}_i^{(3)} = \sqrt{I_3} c_i^{(3)} \, , \quad \bar{c}_i^{(2)} = \sqrt{I_2} c_i^{(2)} \, , \quad \bar{c}_i^{(1)} = \sqrt{I_1} c_i^{(1)} \, , \tag{3.7}$$

and furthermore

$$\phi_i^{(3)} = \arg(c_i^{(3)}) \, , \quad \phi_i^{(2)} = \arg(c_i^{(2)}) \, , \quad \phi_i^{(1)} = \arg(c_i^{(1)}) \, , \tag{3.8}$$

for $i = 1, 2, 3$. In the limit of eq. (3.5), and using that the observed mixing angle θ_{13} is small, the mixing angles of the PMNS matrix (using the standard PDG parameterisation [16]) are then given as

$$\tan\theta_{23} \approx \frac{|\bar{c}_2^{(3)}|}{|\bar{c}_3^{(3)}|} \tag{3.9}$$

$$\tan\theta_{12} \approx \frac{|\bar{c}_1^{(2)}|}{c_{23}|\bar{c}_2^{(2)}|\cos\tilde{\phi}_2 - s_{23}|\bar{c}_3^{(2)}|\cos\tilde{\phi}_3} \tag{3.10}$$

$$\theta_{13} \approx e^{i(\tilde{\phi}+\phi_1^{(2)}-\phi_2^{(3)})} \frac{|\bar{c}_1^{(2)}|(\bar{c}_2^{(3)*}\bar{c}_2^{(2)} + \bar{c}_3^{(3)*}\bar{c}_3^{(2)})}{[|\bar{c}_2^{(3)}|^2 + |\bar{c}_3^{(3)}|^2]^{\frac{3}{2}}} + e^{i(\tilde{\phi}+\phi_1^{(3)}-\phi_2^{(3)})} \frac{|\bar{c}_1^{(3)}|}{\sqrt{|\bar{c}_2^{(3)}|^2 + |\bar{c}_3^{(3)}|^2}} \, , \tag{3.11}$$

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	Best-fit value	Range	C.L.
θ_{12} [$^\circ$]	33.2	$29.3 - 39.2$	99% (3σ)
θ_{23} [$^\circ$]	45.0	$35.7 - 55.6$	99% (3σ)
θ_{13} [$^\circ$]	–	$0.0 - 11.5$	99% (3σ)
Δm^2_{21} [eV^2]	$7.9 \cdot 10^{-5}$	$7.1 \cdot 10^{-5} - 8.9 \cdot 10^{-5}$	99% (3σ)
$\lvert\Delta m^2_{31}\rvert$ [eV^2]	$2.6 \cdot 10^{-3}$	$2.0 \cdot 10^{-3} - 3.2 \cdot 10^{-3}$	99% (3σ)

Table 1: Experimental results for the neutrino mixing angles and mass squared differences, taken from the recent global fit of ref. [18] to the present neutrino oscillation data.

where we have defined

$$\tilde{\phi}_2 = \phi_2^{(2)} - \phi_1^{(2)} - \tilde{\phi} + \delta \,, \tag{3.12}$$

$$\tilde{\phi}_3 = \phi_3^{(2)} - \phi_2^{(2)} + \phi_2^{(3)} - \phi_3^{(3)} - \tilde{\phi} + \delta \,. \tag{3.13}$$

δ and $\tilde{\phi}$ are determined from the condition/convention that $\tan\theta_{12}$ and θ_{13} are real and positive, respectively. Under the above conditions, the neutrino masses are given by

$$m_3 = (|\overline{c}_2^{(3)}|^2 + |\overline{c}_3^{(3)}|^2) \,, \tag{3.14}$$

$$m_2 = \frac{|\overline{c}_1^{(2)}|^2}{s_{12}} \,, \tag{3.15}$$

$$m_1 = O(|\overline{c}^{(1)}|^2) \,. \tag{3.16}$$

We note that from a technical point of view, the procedure which has been used for extracting the neutrino parameters is equivalent to the one for see-saw models of neutrino masses with sequential right-handed neutrino dominance [17]. However, it is applied here in a different physical context, namely that of neutrino masses from string theory instantons which generate the Weinberg operator.

Let us now turn to the conditions for consistency with experiment. The present experimental status is summarised in table 1. We see that under the "sequential dominance" assumptions of eq. (3.5) the following general conditions are imposed on the parameters $c_i^{(r)}$:

- Large, nearly maximal, mixing $\theta_{23} \approx \pi/4$ implies that $|c_2^{(3)}| \simeq |c_3^{(3)}|$.
- Large (but non-maximal) θ_{12} implies that $|c_1^{(2)}| \simeq |c_2^{(2)}| \simeq |c_3^{(2)}|$, or at least that $|c_1^{(2)}|$ and either $|c_2^{(2)}|$ or $|c_3^{(2)}|$ are of the same order.
- Small θ_{13} requires that $|c_1^{(3)}|/|c_3^{(3)}|$ is small.

Generically, coefficients $c_i^{(r)}$ of $\mathcal{O}(1)$ are a typical expectation in the present scheme. This means, large mixings are not only easy to accommodate, but are even expected in the considered scenario. However, the explicit values depend on the details of the model, and small (or even vanishing) values for the $c_a^{(r)}$ amplitudes may emerge in particular

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examples. The condition $|c_1^{(3)}| \ll |c_3^{(3)}|$ may thus give us information/constraints on which string constructions may be fully successful in describing the neutrino data.

For a hierarchical neutrino spectrum, the conditions of eq. (3.5) imply that the particular parameters $c_i^{(1)}$ (corresponding to the most suppressed instanton effect) do not play a significant role for the mixing angles. In fact, only two instantons are required to give masses m_2 and m_3 to two linear combination of neutrinos fields, while one of the neutrinos could remain massless. The remaining constraint is that the two instanton contributions proportional to e^{-S_2} and e^{-S_3} have to generate neutrino masses $m_2 \approx \sqrt{\Delta m_{21}^2}$ and $m_3 \approx \sqrt{|\Delta m_{31}^2|}$.

3.1.2 Normal hierarchy and tri-bimaximal neutrino mixing

One of the most popular proposed structures for neutrino mixing is that of tri-bimaximal mixing [19]. We would like now to study under what conditions the neutrino mass matrix from string theory instantons via the Weinberg operator, i.e. of the form of eq. (3.3), can give rise to tri-bimaximal lepton mixing. Tri-bimaximal lepton mixing is a pattern of neutrino mixing angles postulated by [19], where the PMNS matrix is given by

$$U_{\text{tri}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \tag{3.17}$$

In the standard PDG parameterisation [16], this corresponds to $\theta_{12} = \arcsin(1/\sqrt{3}) \approx 35.3^\circ, \theta_{13} = 0$ and $\theta_{23} = \arcsin(1/\sqrt{2}) = 45^\circ$ in the lepton sector. The PMNS matrix is usually given in the basis where the so-called "unphysical phases" are eliminated by absorbing a global phase factor in the definition of the lepton doublets. Since in our case neutrinos have Majorana masses, the PMNS matrix is multiplied by an additional phase matrix $P_{\text{Maj}} = \text{diag}(e^{i\frac{\alpha_1}{2}}, e^{i\frac{\alpha_2}{2}}, 0)$ from the right, which contains the Majorana phases α_1, α_2. As stated earlier, we assume that the large mixing in the lepton sector originates from the neutrino mass matrix, such that we may treat the small mixings of the charged lepton mass matrix as a perturbation. We will consider the minimal case of two instantons, the minimal number required in order generate two massive neutrinos.

Let us try to find an example for tri-bimaximal mixing of the neutrino mass matrix from instantons. To start with, we may assume a normal hierarchy for the neutrino masses, i.e. $m_1 \ll m_2 \approx \frac{1}{5}m_3$, and set m_1 to zero. Using the expression (3.17) one obtains for the the neutrino mass matrix with tri-bimaximal mixing

$$\begin{aligned} M^{\text{tri}} &= U_{\text{tri}}\, \text{diag}(0, m_2\, e^{i\alpha_2}, m_3)\, U_{\text{tri}}^T \\ &= \frac{m_2\, e^{i\alpha_2}}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}. \end{aligned} \tag{3.18}$$

By comparing this form with eq. (3.3), we see that a possibility to obtain this structure is to identify $I_2 = m_2, I_3 = m_3$ and to choose the coefficients $c_i^{(2)}, c_i^{(3)}$ as

$$(c_1^{(2)}, c_2^{(2)}, c_3^{(2)}) = \frac{1}{\sqrt{3}} e^{i\frac{\alpha_2}{2}} (1,1,1) , \tag{3.19}$$

$$(c_1^{(3)}, c_2^{(3)}, c_3^{(3)}) = \frac{1}{\sqrt{2}}(0,-1,1) . \tag{3.20}$$

We would like to remark that this is one particular possibility, not the most general case. However, with a hierarchy among the neutrino masses and a hierarchy among the instanton contributions, it is suggestive that one instanton generates m_2 and the other one m_3. Note also that the "normalisation" of the "flavour vectors" $c_i^{(r)}$ can be changed to $c^{(r)} \to \mathcal{N} c^{(r)}$ by choosing $I_r = \mathcal{N}^{-2} m_r$ instead of $I_r = m_r$.

We see that obtaining precisely the structure of tri-bimaximal mixing requires the flavour vectors $c_a^{(2)}, c_a^{(3)}$ to align along specific flavour directions.[4] On the other hand obtaining masses and mixings compatible with experiment require much milder constraints on the flavour vectors, as we discussed in the previous sections.

3.1.3 The inverse hierarchy case

Experimentally, two possibilities for the ordering of the neutrino masses are allowed: The so-called normal ordering where $m_1 < m_2 < m_3$, and the so-called inverse ordering where $m_3 < m_1 < m_2$. If in the latter case $m_3 \ll m_1 \lesssim m_2$, the neutrino spectrum is called inverse hierarchical. String theory instantons can in principle also give rise to this scenario. However, we have to keep in mind that the splitting between m_1 and m_2 is very small, and that it would have to be explained why $m_1 \approx m_2$. Within the string instanton point of view, this would require the presence of two instantons $D2_1$, $D2_2$ with approximately the same actions $S^{(1)}$,$S^{(2)}$ but with very different flavour vectors $c_a^{(1)}, c_a^{(2)}$. Since the action is given essentially by the size of the wrapped 3-volume in extra dimensions and the latter are expected to be generically different, a certain amount of fine-tuning would be required. Different values for the different actions $S^{(r)}$, typically leading to some hierarchy seems more generic. Aiming at completeness, we will nevertheless consider the inverse hierarchy case as well.

Examples of patterns for the relevant coefficients $c_i^{(1)}$ and $c_i^{(2)}$ in this case can be found easily following the strategy used in the above subsection. Since general analytic formulae are rather lengthy, we will focus on a particular example here, noting that many variations and alternative patterns are possible and allowed by the experimental data. As above, we consider the example of tri-bimaximal mixing since approximate tri-bimaximality is well compatible with the present experimental data. For the inverse hierarchy case, in principle tri-bimaximal mixing (and other patterns of neutrino mixing angles compatible with experiment) could be realised as well. Setting $m_3 = 0$, the neutrino mass matrix with

[4] Note in particular that in order to exactly reproduce the tri-bimaximal mixing matrix there should be three instantons with flavour vectors $c_a^{(r)}$ aligning along the Cartan subalgebra generators of a U(3) group.

tri-bimaximal mixing has the form

$$
\begin{aligned}
M^{\rm tri} &= U_{\rm tri}\,{\rm diag}(m_1\,e^{i\alpha_1}, m_2\,e^{i\alpha_2}, 0)\,U_{\rm tri}^T \\
&= \frac{m_1\,e^{i\alpha_1}}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2\,e^{i\alpha_2}}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\,,
\end{aligned}
\tag{3.21}
$$

and only two string instantons are required (in the most minimal case) to give neutrino masses m_1 and m_2. Again, comparing with eq. (3.3) we see that a possible choice is $I_1 = m_1, I_2 = m_2$ (with $I_1 \simeq I_2$) and

$$
(c_1^{(1)}, c_2^{(1)}, c_3^{(1)}) = \frac{1}{\sqrt{6}}\,e^{i\frac{\alpha_1}{2}}\,(-2, 1, 1)\,, \tag{3.22}
$$

$$
(c_1^{(2)}, c_2^{(2)}, c_3^{(2)}) = \frac{1}{\sqrt{3}}\,e^{i\frac{\alpha_2}{2}}\,(1, 1, 1)\,. \tag{3.23}
$$

3.1.4 Quasi-degenerate neutrino spectrum

We now turn to the general case, which includes the cases of quasi-degenerate (or partially-degenerate) neutrino masses with m_1, m_2, m_3 non-zero and with typically two of them being nearly degenerate in mass. In our scheme one can in principle accommodate this scenario as well. However, now the splitting between m_1, m_2 and m_3 are very small, and this almost degeneracy of the mass eigenvalues would have to be explained. Explicitly, the masses have to satisfy the experimental constraints, i.e. $m_2^2 - m_1^2 \approx 7.9 \times 10^{-5}\,{\rm eV}^2$, $|m_3^2 - m_1^2| \approx 2.6 \times 10^{-3}\,{\rm eV}^2$ (c.f. table 1), while $m_1 \approx m_2 \approx m_3$ are much larger than the mass splitting. From the string instanton point of view, this would require again having three different instanton with almost identical action but very different flavour vectors. Although possible such situation would require some fine tuning and is generically unexpected.

In order to give an example for a pattern of $c_i^{(r)}$ compatible with the experimentally found mixing angles, let us consider again the concrete example of tri-bimaximal mixing. Tri-bimaximal mixing for quasi-degenerate neutrinos could be realised with three instantons with $c^{(1)}, c^{(2)}, c^{(3)}$, chosen as

$$
(c_1^{(1)}, c_2^{(1)}, c_3^{(1)}) = \frac{1}{\sqrt{6}}\,e^{i\frac{\alpha_1}{2}}\,(-2, 1, 1)\,, \tag{3.24}
$$

$$
(c_1^{(2)}, c_2^{(2)}, c_3^{(2)}) = \frac{1}{\sqrt{3}}\,e^{i\frac{\alpha_2}{2}}\,(1, 1, 1)\,, \tag{3.25}
$$

$$
(c_1^{(3)}, c_2^{(3)}, c_3^{(3)}) = \frac{1}{\sqrt{2}}(0, -1, 1)\,, \tag{3.26}
$$

and with $I_1 = m_1, I_2 = m_2, I_3 = m_3$ (and $I_1 \simeq I_2 \simeq I_3$).

3.2 The see-saw case

Up to now we have considered the case in which the leading contribution to the observed neutrino masses comes from the Weinberg operator. Let us consider now the inverse case in which the see-saw mechanism gives the leading contribution to neutrino masses. In general, the see-saw contribution to the neutrino mass matrix can depend significantly on the structure of the neutrino Yukawa matrix h_D, leading to a large variety of possible patterns of h_D and M^R_{ab} consistent with the experimental neutrino data (assuming again small charged lepton mixing, as before).

Obtaining analytic formulae for the most general see-saw case is difficult. In the following, we discuss the special case in which only small mixing stems from the neutrino Yukawa matrix h_D. Explicitly, we will consider the limit that h_D is diagonal, i.e.

$$h_D = \mathrm{diag}(y_e^{(\nu)}, y_\mu^{(\nu)}, y_\tau^{(\nu)}) \ . \tag{3.27}$$

Small mixing induced by h_D may be treated as a perturbation and can be included in a straightforward way. We will furthermore assume that the see-saw contribution dominates the neutrino mass matrix.

The neutrino mass matrix is then given by

$$M^L_{ab} = \left(\sum_r \frac{d_a^{(r)}}{(h_D^T)_{aa}} \frac{d_b^{(r)}}{(h_D)_{bb}} \widetilde{I}_r^{-1} \right)^{-1} \ , \tag{3.28}$$

where we have introduced

$$\widetilde{I}_r = \frac{\langle \bar{H} \rangle^2}{2M_s} \frac{1}{e^{-S_r}} \ . \tag{3.29}$$

and which defines the quantity

$$\widetilde{M}^L_{ab} := (M^L_{ab})^{-1} = \sum_r \frac{d_a^{(r)}}{(h_D^T)_{aa}} \frac{d_b^{(r)}}{(h_D)_{bb}} \widetilde{I}_r^{-1} \ . \tag{3.30}$$

The indices of the matrix $\widetilde{M}^L_{ab}$ must be understood as those coming from the numbers $d_a^{(r)}$ and $d_b^{(r)}$, while those of the matrix M^L_{ab} must be calculated as in the usual matrix calculus.

As an example, we now discuss how to choose the coefficients $d_a^{(r)}$ in order to realise tri-bimaximal neutrino mixing. We note that this procedure can be readily generalised to any other desired pattern of neutrino mixings. We first observe that if we find $d_a^{(r)}$ such that U_{tri} diagonalises $\widetilde{M}^L_{ab}$, then also M^L_{ab} has tri-bimaximal form,

$$U_{\mathrm{tri}}^T \, \widetilde{M}^L \, U_{\mathrm{tri}} = \mathrm{diag}(\tfrac{1}{m_1}, \tfrac{1}{m_2}, \tfrac{1}{m_3}) \ \Rightarrow \ U_{\mathrm{tri}}^T \, (\widetilde{M}^L)^{-1} \, U_{\mathrm{tri}} = \mathrm{diag}(m_1, m_2, m_3) \ , \tag{3.31}$$

since U_{tri} is orthogonal. The form of $\widetilde{M}^L$ required to realise tri-bimaximal mixing is thus given by

$$\begin{aligned} \widetilde{M}^L &= U_{\mathrm{tri}} \, \mathrm{diag}\left(\frac{1}{m_1 \, e^{i\alpha_1}}, \frac{1}{m_2 \, e^{i\alpha_2}}, \frac{1}{m_3} \right) U_{\mathrm{tri}}^T \\ & \frac{1}{6\, m_1 \, e^{i\alpha_1}} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{1}{3\, m_2 \, e^{i\alpha_2}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{1}{2\, m_3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} . \end{aligned} \tag{3.32}$$

A possible choice for the $d_a^{(r)}$ is therefore (analogous to the Weinberg operator cases) $\widetilde{I}_1 = m_1, \widetilde{I}_2 = m_2, \widetilde{I}_3 = m_3$ and

$$\left(\frac{d_1^{(1)}}{y_e^{(\nu)}}, \frac{d_2^{(1)}}{y_\mu^{(\nu)}}, \frac{d_3^{(1)}}{y_\tau^{(\nu)}}\right) = \frac{1}{\sqrt{6}} e^{-i\frac{\alpha_1}{2}} (-2, 1, 1) , \tag{3.33}$$

$$\left(\frac{d_1^{(2)}}{y_e^{(\nu)}}, \frac{d_2^{(2)}}{y_\mu^{(\nu)}}, \frac{d_3^{(2)}}{y_\tau^{(\nu)}}\right) = \frac{1}{\sqrt{3}} e^{-i\frac{\alpha_2}{2}} (1, 1, 1) , \tag{3.34}$$

$$\left(\frac{d_1^{(3)}}{y_e^{(\nu)}}, \frac{d_2^{(3)}}{y_\mu^{(\nu)}}, \frac{d_3^{(3)}}{y_\tau^{(\nu)}}\right) = \frac{1}{\sqrt{2}}(0, -1, 1) . \tag{3.35}$$

As discussed for the Weinberg operator case, only two of the right-handed neutrinos are relevant in the limit of the normal and inverse hierarchy cases.

We see that if $(h_D)_{aa} = y_a^{(\nu)}$, $a = e, \mu, \tau$, are hierarchical, then also the $d_a^{(r)}$ (for all r) would have to have a very similar hierarchical structure, in order to generate large neutrino mixing. Although possible in principle, this would be a significant constraint on models. On the other hand, with $(h_D)_{aa} = y_a^{(\nu)}$ being all of the same order, the conditions of eq. (3.33) could be comparatively easier to satisfy and large neutrino mixing angles would be a generic expectation.

A different possibility would of course be that only small mixing is induced by M^R and that large mixing originates from h_D. In this case, we recover the known conditions on h_D and M^R discussed extensively in the literature on conventional see-saw models [20].

3.3 The general case: Weinberg operator and see-saw

More generally, instantons may generate the Weinberg operator for neutrino masses, which provides a direct mass term for (some of) the three light neutrinos after EW symmetry breaking, as well as the Majorana mass matrix for the right-handed neutrinos. The full neutrino mass matrix M has dimension 6×6,

$$M = \begin{pmatrix} M^L & v h_D^T \\ v h_D & M^R \end{pmatrix} . \tag{3.36}$$

Beyond the discussion of the previous sections, there is the possibility that the contributions from the see-saw mechanism and from the Weinberg operator both contribute with similar strength to the mass matrix of the light neutrinos. For example, one may have the case that one of the contributions generates the dominant term in eq. (3.18), while the other generates the sub-dominant one.

Finally, it is also possible in principle that some of the right-handed neutrinos could obtain rather small masses, such that there are more than three light neutrino mass eigenstates (or right-handed neutrinos close to the EW scale). In specific string models, all ingredients of the neutrino mass matrix M are (in principle) computable. If such more unconventional scenarios should appear as predictions, a more careful analysis of constraints from oscillation experiments, electroweak decays and cosmological observations would be required to test consistency of such a string model with respect to the neutrino sector data.

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4. Conclusions and outlook

In this paper we have explored the structure of neutrino masses originating from certain string theory instanton effects recently pointed out in the literature [1, 2, 4]. They appear in string compactifications in which the SM group is extended by a $\mathrm{U}(1)_{B-L}$ getting a Stuckelberg mass. Our analysis has concentrated in the simplest class of such instantons with a Sp(2) Chan-Paton symmetry. These instantons lead to a certain flavour-factorised form for both, the ν_R mass matrix and the Weinberg operator. A hierarchy of neutrino masses naturally appears from the different values of the actions for the different contributing instantons. For the case that the Weinberg operator gives rise to the leading contribution to neutrino masses, we have shown how one can reproduce the experimental patterns for neutrino masses and mixings under not very restrictive conditions on the instanton amplitudes $c_a^{(r)}$. For particular directions of these flavour vectors $c_a^{(r)}$ one may reproduce, for example, the structure of tri-bimaximal mixing. This is true both for normal and inverted hierarchy cases, although the latter seems more unlikely within the present scheme. In the opposite case in which the see-saw mechanism gives rise to the leading contribution, the structure of neutrino masses depends strongly on the form of the Dirac neutrino mass matrix. In a simplified situation with a diagonal Dirac mass matrix one can obtain, e.g., tri-bimaximal mixing if the flavour vector coefficients $d_a^{(r)}$ align along certain flavour directions. The often assumed situation with a diagonal ν_R mass matrix and mixing originated in the Dirac sector is also possible.

A number of extra possibilities should be explored. Other classes of string instantons [4] with Chan-Paton symmetries $O(1)$ and $\mathrm{U}(1)$ in general do not lead to a factorised flavour dependence of both ν_R masses and Weinberg operator. It would be interesting to explore the phenomenological possibilities for these other classes of instantons. From the string model building point of view, it would be important to learn more about the structure and flavour dependence of the flavour vectors $c_a^{(r)}$ and $d_a^{(r)}$ in particular string compactifications. To do that a search for models with an extra $\mathrm{U}(1)_{B-L}$ gauge boson which becomes massive through a Stuckelberg is required. Getting a neutrino spectrum consistent with experimental constraints would be a new important test of string models.

One assumption we have made is that the contribution to the leptonic mixing matrix from the mass matrix of the charged leptons is small. This condition is satisfied in many well motivated phenomenological models, where there is only small mixing in the mass matrices of quarks as well as charged leptons. We note however that in general, large mixing can as well stem from the charged lepton sector (see e.g. [21]) or from a combination of both, neutrino and charged lepton contributions. The conditions derived in this letter can be readily generalised to these scenarios as well. For the case of small mixing from the charged lepton sector, the charged lepton contributions can be treated as corrections to the neutrino mixing angles and CP phases (see e.g. [22]). The general conditions for consistency with neutrino data do not change due these small corrections. In the case that the charged lepton mixing matrix is CKM-like, i.e., small and dominated by a 1-2 mixing, and for small 1-3 mixing in the neutrino mass matrix, the neutrino mixing sum rule $\theta_{12} - \theta_{13}\cos(\delta) \approx \theta_{12}^{\nu}$ [22 – 24] holds between the measurable PMNS parameters $\theta_{12}, \theta_{13}, \delta$

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and the theoretical prediction for the 1-2 mixing angle θ_{12}^{ν} from the diagonalisation of the neutrino mass matrix. Thus, the prediction for the neutrino mixing angle θ_{12}^{ν}, which is directly connected to the string instantons, can be tested by precisely measuring θ_{13}, θ_{12} and δ in future neutrino experiments [25].

Regarding leptogenesis, there is one conceptually interesting fact: All of leptogenesis would have its origin in instantons. ν_R masses would come from string instantons, and the transformation of lepton into baryon asymmetry would be due to $\mathrm{SU}(2)_L$ gauge instantons. Leptogenesis would proceed via the out-of-equilibrium decay of the right-handed (s)neutrinos, and in the general case both, the other right-handed neutrinos as well as the Weinberg operator would contribute to the decay asymmetries proportional to their contribution to the neutrino mass matrix [26]. In this respect it is worth noting that the flavour vectors $c_a^{(r)}, d_a^{(r)}$ are in general complex and so will be the generated neutrino mass matrices. It would be interesting to explore in more detail whether (semi-)realistic string constructions consistent with the low energy neutrino data could also give rise to successful baryogenesis via leptogenesis.

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